Goldstein

1.8 (correction.) $L' = L + \frac{d}{dt} F(q_1, \cdots, q_n, t)$

We want to show

$$\frac{d}{dq_k}\left(\frac{d}{dt}F\right) = \frac{d}{dt}\frac{d}{d\dot{q}_k}\left(\frac{d}{dt}F\right)$$

$\frac{d}{dt}F = \frac{\partial F}{\partial q_i}\dot{q}_i$ (summation implied)

$$\Rightarrow \quad \frac{d}{dq_k}\left(\frac{d}{dt}F\right) = \boxed{\frac{d}{dq_k}\left(\frac{\partial F}{\partial q_i}\dot{q}_i\right)}$$

$$\frac{d}{dt}\frac{d}{d\dot{q}_k}\left(\frac{d}{dt}F\right) = \frac{d}{dt}\frac{d}{d\dot{q}_k}\left(\frac{\partial F}{\partial q_i}\dot{q}_i\right)$$

$$= \frac{d}{dt}\left(\frac{\partial F}{\partial q_k}\right)$$

$$= \frac{d}{dq_k}\left(\frac{d}{dt}F\right)$$

$$= \boxed{\frac{d}{dq_k}\left(\frac{\partial}{\partial q_i}\dot{q}_i\right)}$$

equal

equal

Davidson Chang

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